Clutches and Brakes الفواصل والمكابح

17-1 INTRODUCTION

In the operation of a friction clutch during engagement, the initial condition is that the driving member is moving and the driven member is at rest; in the final condition both members are moving at the same speed, i.e., with no relative motion. In the operation of a brake during braking, the initial condition is that one member, such as the brake wheel or drum, is moving and the braking member is stationary; in the final condition both members are at rest and have no relative motion.

It is thus apparent that the principle of operation of both a friction clutch during engagement and of a friction brake during braking is to bring two members having relative motion to the state of no relative motion, The operation of a clutch is therefore essentially the same as that of a brake; however, there are structural differences in the two units because of control requirements and the necessity for providing for heat absorption or dissipation in brakes.

17-2 PLATE CLUTCHES AND BRAKES

Plate clutch shown in Fig.17-1,(*A*, driver; *B*, driven plate; *C*, actuator), the part *A* is connected to the driving unit (engine or motor), and the driving plate *B* are splined to the shaft.

Force analysis In Fig. 17-2 there is shown a friction disk with outer and inner diameters, *D* and *d*, respectively. The mating disk, produces a pressure *p* on the surface because of the axial load *P*.

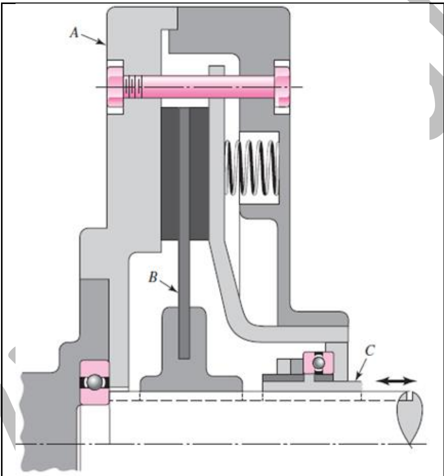


Figure 17–1 A single-plate clutch

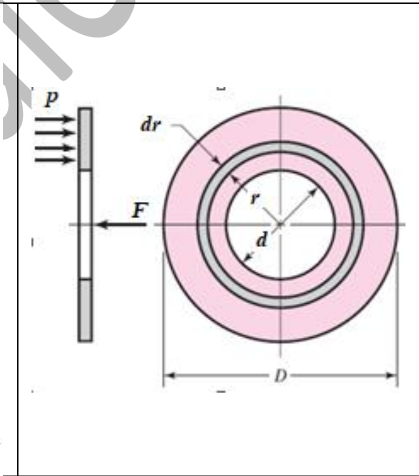


Figure 17–2

Elementary surface area $dA = 2\pi r dr$

Normal force on $dA = p2\pi r dr$

Frictional force on $dA = \mu p2\pi r dr$

where p =surface pressure

μ =coefficient of friction, assumed as constant. Therefore

$$P = 2\pi \int_{d/2}^{D/2} pr\, dr \qquad (a)$$

and $$T = 2\pi f \int_{d/2}^{D/2} pr^2\, dr \qquad (b)$$

Before Eqs. (*a*) and (*b*) may be integrated, it is necessary to make an assumption regarding the distribution of pressure. Two cases will be considered:

1- **Uniform pressure**. For new clutches and rigid mountings, the pressure may be assumed to be uniformly distributed over the contact area; hence, *p* may be regarded as constant.

Therefore, from Eq. (*b*),

$$T = \frac{\pi \mu p}{12}(D^3 - d^3) \qquad (c)$$

By eliminating *p* from Eqs. (c) and (d) and solving, the torque for one pair of friction surfaces in contact is

$$\boldsymbol{T = \frac{\mu P}{3}\left(\frac{D^3 - d^3}{D^2 - d^2}\right) = \frac{2\mu P}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) = \mu P r_m} \qquad (\mathbf{17-1}\boldsymbol{a})$$

$$Where\ r_m = \frac{1}{3}\left(\frac{D^3 - d^3}{D^2 - d^2}\right) = \frac{2}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right)$$

The total torque for multi-disc clutch

$$\boldsymbol{T = \frac{\mu P n}{3}\left(\frac{D^3 - d^3}{D^2 - d^2}\right) = \frac{2\mu P n}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) = \mu P r_m n} \qquad (\mathbf{17-1}\boldsymbol{b})$$

Where $\boldsymbol{n}$ is the number of pairs of contact surfaces
and, from Eq. (*a*), the actuating force is

$$\boldsymbol{P = \frac{\pi p}{4}(D^2 - d^2) = \pi p\left(r_o^2 - r_i^2\right)} \qquad (\mathbf{17-1}\boldsymbol{c})$$

2- **Uniform axial wear**. An inspection of worn clutch plates reveals that the plates are not of uniform thickness. Uniformity of wear, it is necessary to make an assumption regarding wear.

- The rate of wear depends on
  1- the friction force at the surfaces in contact, which depends on the pressure, $w \propto \mu p$
  2- the rubbing velocity $n \propto 2\pi r$
- Thus, the wear on an element of area in one revolution depends on the product of μp and the distance $2\pi r$

$$\boldsymbol{w \propto \mu p \times 2\pi r}$$

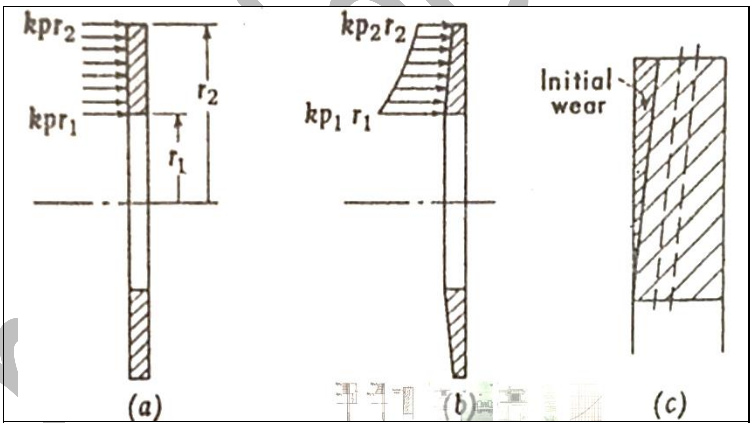


Fig. 17-3

If the coefficient of friction is assumed as constant,

The wear is proportional to the product pr.

$$w \propto pr \qquad \text{or} \qquad w = kpr$$

k is the constant of proportionality

When the equilibrium condition is reached,

$$\left.\begin{array}{c} \boldsymbol{w = kpr} \\ \boldsymbol{C = pr} \\ \boldsymbol{p_{max} r_i = p_{min} r_o = C} \\ \boldsymbol{p = \dfrac{C}{r}} \end{array}\right\} \ldots\ldots\ldots\ldots (\boldsymbol{c})$$

By substituting the value of p from Eq. (c) into Eqs. (a) and (b), integrating, and eliminating the constant C, the equation for the torque becomes:

$$\boldsymbol{T = \mu \frac{P}{4}(D + d) = \mu P r_m} \qquad (\mathbf{17-2})$$

where T is the torque for one pair of friction surfaces in contact

$$r_m = \frac{D + d}{4} = \frac{\boldsymbol{r_i + r_o}}{2}$$

the equation for the actuating force becomes:

$$\boldsymbol{P = \frac{\pi p d}{2}(D - d) = \pi p_{max} r_i (D - d) = 2\pi p_{max} r_i (r_o - r_i) = 2\pi p_{min} r_o (r_o - r_i)} \qquad (\mathbf{17-3})$$

هذه المعادلة مفيدة في إيجاد أكبر ضغط تتعرض له حشوة الفاصل او المكبح لمقارنته مع خصائص الحشوة من الجداول الخاصة بها كما في الجدول الاتي:

Table 17-1 Coefficients of friction and allowable pressures

Material	Friction Coefficient Wet	Friction Coefficient Dry	Max. Temperature °F	Max. Temperature °C	Max. Pressure psi	Max. Pressure kPa
Cast iron on cast iron	0.05	0.15–0.20	600	320	150–250	1000–1750
Powdered metal* on cast iron	0.05–0.1	0.1–0.4	1000	540	150	1000
Powdered metal* on hard steel	0.05–0.1	0.1–0.3	1000	540	300	2100
Wood on steel or cast iron	0.16	0.2–0.35	300	150	60–90	400–620
Leather on steel or cast iron	0.12	0.3–0.5	200	100	10–40	70–280
Cork on steel or cast iron	0.15–0.25	0.3–0.5	200	100	8–14	50–100
Felt on steel or cast iron	0.18	0.22	280	140	5–10	35–70
Woven asbestos* on steel or cast iron	0.1–0.2	0.3–0.6	350–500	175–260	50–100	350–700
Molded asbestos* on steel or cast iron	0.08–0.12	0.2–0.5	500	260	50–150	350–1000
Impregnated asbestos* on steel or cast iron	0.12	0.32	500–750	260–400	150	1000
Carbon graphite on steel	0.05–0.1	0.25	700–1000	370–540	300	2100

*The friction coefficient can be maintained with ±5 percent for specific materials in this group.

- In Eqns. (17-1) and (17-2) the terms $\frac{1}{3}\frac{D^3-d^3}{D^2-d^2}$ and $\frac{1}{4}(D+d)$ may be called the friction radii.
- A comparison of these terms shows that the friction radius for new clutches is slightly larger than for worn-in clutches,
- the percentage difference may be expressed in terms of the ratio D/d, as shown in Fig. 17-4.
- The ratio $D/d \cong 1.5$ for industrial clutches and brakes and for automotive clutches.
- For this proportion the difference in the two equations is very low and is much smaller than the variation in the value for the coefficient of friction; hence on the basis of accuracy the choice between the two equations is unimportant.
-
- Eq. (17-2) gives values for the axial force which are on the side of safety, it applies to most of the life of the plates, and it can be more directly related to space limitations during design and is therefore recommended for use rather than the uniform-pressure equation.

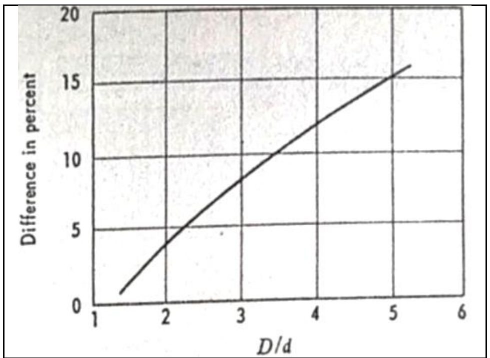


Fig. 17-4 Comparison of Eqs. (17-1) and (17-2).

Multidisk clutch Where large torques must be transmitted, a multidisk clutch, as shown in the figure below, may be used to limit the operating force. The torque transmitted by this clutch may be determined by:

$$T = \frac{\mu P n}{4}(D+d) = \frac{\mu P n}{2}(r_o - r_i) \quad (17-4)$$

n: the number of pairs of friction surfaces. n may be found using the following relation:

$$n = n_1 + n_2 - 1$$

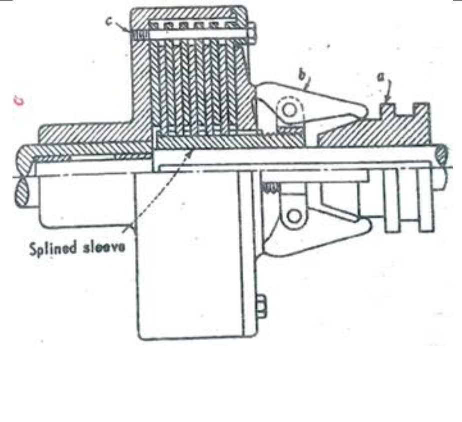
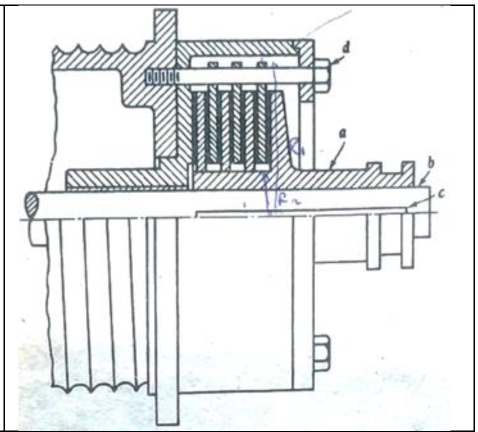
n_1 is the number of driving discs

n_2 is the number of driving discs

Multidisk brakes are used to a limited extent in some installations where speed control is necessary. In the figure, there is shown a multidisc brake used with a hoisting drum. the shaft b by means of the feather key c, thus causing a to rotate with the shaft but permitting it to slide axially during engagement and disengagement. When it other set slides axially on the bolts d but rotates with the drum.

is desired to raise the load, a force on the operating lever slides the member *a* to the left and connects it with the drum, and the unit acts as a clutch. The shaft is prevented from rotating backward by a ratchet. In order to lower the load, the operating force is decreased, which allows the load to be lowered.

Single-disk brakes are presently developed for front-wheel automotive

	
Multidisc clutch.	**Multidisk brake.**

Example(1): Design a single-plate friction disc clutch to transmit 25 kW at 1500 rpm. The outer diameter of the friction disc is 250 mm, and the inner diameter is 150 mm. Assume:

1. **Uniform pressure distribution** and
2. **Uniform wear distribution**.

The coefficient of friction is 0.3, and the allowable pressure intensity is 0.8 MPa. Determine the axial force required to engage the clutch and the number of friction surfaces needed for both cases.

Given Data:

- P=25 kW , N=1500 rpm, D =250 mm, d=150 mm, μ=0.3, p=0.8 MPa

Step 1: Calculate the Torque to be Transmitted

The torque T is calculated using the power and speed:

$$T = \frac{60H}{2\pi n} = \frac{25 \times 10^3 \times 60}{2\pi \times 1500} = 159 \text{ N m}$$

Step 2: Calculate the Mean Radius

The mean radius R_m is calculated differently for uniform pressure and uniform wear cases.

For Uniform Pressure:

$$r_m = \frac{1}{3}\left(\frac{D^3 - d^3}{D^2 - d^2}\right) = \frac{2}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right)$$

$$r_o = \frac{D_o}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

$$r_i = \frac{D_i}{2} = 75 \text{ mm} = 0.075 \text{ m}$$

$$r_m = \frac{2}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) = \frac{2}{3}\left(\frac{0.125^3 - 0.075^3}{0.125^2 - 0.075^2}\right) = 0.102\ m$$

For Uniform Wear:

$$r_m = \frac{D + d}{4} = \frac{0.25 + .15}{4} = 0.1\ m$$

Step 3: Calculate the Axial Force Required

For Uniform Pressure:

The axial force P is given by:

$T=\mu \cdot P \cdot r_m \cdot n$

$P=T/\mu \cdot r_m \cdot n$

For a single-plate clutch, n=2:

P=159/0.3×0.102×2=159.15/0.0612 =2,598 N

For Uniform Wear:

The axial force P is given by:

$T=\mu \cdot P \cdot r_m \cdot n$

$P=T/\mu \cdot r_m \cdot n$

For a single-plate clutch, $n=2$:

$P=159/0.3\times0.1\times2=159/0.06=2650$

Step 4: Verify the Axial Force with Allowable Pressure

For Uniform Pressure:

The axial force P is also given by:

$$P = p \cdot A$$

Where A is the area of the friction surface:

$$A = \pi \left(r_o^2 - r_i^2\right)$$

$$A = \pi \left((0.125)^2 - (0.075)^2\right)$$

$$= 0.031416 \text{ m}^2$$

Now, calculate P:

$$P = 0.8 \times 10^6 \times 0.031416 = 25133\ N$$

For Uniform Wear:

The axial force F is given by

$$P = 2\pi p_{max} r_i (r_o - r_i)$$

$$= 2\pi \times 0.8 \times 10^6 \times 0.075 \times (0.125 - 0.075)$$

$$P = 18850\ N$$

Step 5: Compare the Axial Forces

For Uniform Pressure:

- Axial force from torque calculation: P=2,600.5 N
- Axial force from allowable pressure: P=25,133 N

For Uniform Wear:

- Axial force from torque calculation: P=2,652.5 N
- Axial force from allowable pressure: P=18,850 N

In both cases, the axial force required to transmit the torque is much less than the axial force allowed by the pressure limit, ensuring the clutch design is safe.

Step 6: Determine the Number of Friction Surfaces

For a single-plate clutch, the number of friction surfaces is $n = 2$. Since the axial force required is well within the allowable limit, no additional friction surfaces are needed.

Final Answer:

Uniform Pressure Case:

- **Axial Force Required:** P=2,598 N
- **Number of Friction Surfaces:** n=2

Uniform Wear Case:

- **Axial Force Required:** P=2,650 N
- **Number of Friction Surfaces:** n=2

Both cases show that the clutch design is safe and can transmit the required torque.

Example 24.1. *Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : $W = 4$ kN = 4000 N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\ p_{max}$$

We also know that total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 50\ p_{max}\,(100 - 50) = 15\ 710\ p_{max}$$

$$\therefore \quad p_{max} = 4000 / 15\ 710 = 0.2546 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore,

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\ p_{min}$$

We know that the total force on the contact surface (W),

$$4000 = 2\pi C (r_1 - r_2) = 2\pi \times 100\, p_{min} (100 - 50) = 31\,420\, p_{min}$$

$\therefore$ $p_{min} = 4000 / 31\,420 = 0.1273$ N/mm^2 **Ans.**

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$= \frac{4000}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \text{ Ans.}$$

Example 24.2. *A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4.*

(a) Assuming a uniform pressure of 0.17 N/mm^2; determine the inner diameter of the friction surfaces.

(b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached.

Solution. Given : $P = 110$ kW $= 110 \times 10^3$ W ; $N = 1250$ r.p.m. ; $d_1 = 300$ mm or $r_1 = 150$ mm ; $\mu = 0.4$; $p = 0.17$ N/mm^2

(a) Inner diameter of the friction surfaces

Let d_2 = Inner diameter of the contact or friction surfaces, and

r_2 = Inner radius of the contact or friction surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{110 \times 10^3 \times 60}{2\pi \times 1250} = 840 \text{ N-m}$$

$$= 840 \times 10^3 \text{ N-mm}$$

Axial thrust with which the contact surfaces are held together,

$$W = \text{Pressure} \times \text{Area} = p \times \pi [(r_1)^2 - (r_2)^2]$$

$$= 0.17 \times \pi [(150)^2 - (r_2)^2] = 0.534 [(150)^2 - (r_2)^2] \quad ...(i)$$

and mean radius of the contact surface for uniform pressure conditions,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right]$$

$\therefore$ Torque transmitted by the clutch (T),

$$840 \times 10^3 = n.\mu.W.R$$

$$= 2 \times 0.4 \times 0.534 [(150)^2 - (r_2)^2] \times \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right] \quad ...(\because n = 2)$$

$$= 0.285 [(150)^3 - (r_2)^3]$$

or $(150)^3 - (r_2)^3 = 840 \times 10^3 / 0.285 = 2.95 \times 10^6$

$\therefore$ $(r_2)^3 = (150)^3 - 2.95 \times 10^6 = 0.425 \times 10^6$ or $r_2 = 75$ mm

and $d_2 = 2r_2 = 2 \times 75 = 150$ mm **Ans.**

(b) Maximum torque transmitted

We know that the axial thrust,

$$W = 0.534\,[(150)^2 - (r_2)^2] \quad \text{... [From equation } (i)]$$
$$= 0.534\,[(150)^2 - (75)^2] = 9011 \text{ N}$$

and mean radius of the contact surfaces for uniform wear conditions,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 75}{2} = 112.5 \text{ mm}$$

∴ Maximum torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.4 \times 9011 \times 112.5 = 811 \times 10^3 \text{ N-mm}$$
$$= 811 \text{ N-m}$$ **Ans.**

Maximum intensity of pressure

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = p_{max} \times 75 \text{ N/mm}$$

We know that the axial thrust (W),

$$9011 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times p_{max} \times 75\,(150 - 75) = 35\,347\,p_{max}$$

∴ $$p_{max} = 9011 / 35\,347 = 0.255 \text{ N/mm}^2$$ **Ans.**

H.W الواجب البيتي

1. A single disc clutch with both sides of the disc effective is used to transmit 10 kW power at 900 r.p.m. The axial pressure is limited to 0.085 N/mm^2. If the external diameter of the friction lining is 1.25 times the internal diameter, find the required dimensions of the friction lining and the axial force exerted by the springs. Assume uniform wear conditions. The coefficient of friction may be taken as 0.3.
[Ans. 132.5 mm ; 106 mm ; 1500 N]

2. A single plate clutch with both sides of the plate effective is required to transmit 25 kW at 1600 r.p.m. The outer diameter of the plate is limited to 300 mm and the intensity of pressure between the plates not to exceed 0.07 N/mm^2. Assuming uniform wear and coefficient of friction 0.3, find the inner diameter of the plates and the axial force necessary to engage the clutch.
[Ans. 90 mm ; 2375 N]

3. Give a complete design analysis of a single plate clutch, with both sides effective, of a vehicle to transmit 22 kW at a speed of 2800 r.p.m. allowing for 25% overload. The pressure intensity is not to exceed 0.08 N/mm^2 and the surface speed at the mean radius is not to exceed 2000 m/min. Take coefficient of friction for the surfaces as 0.35 and the outside diameter of the surfaces is to be 1.5 times the inside diameter. The axial thrust is to be provided by 6 springs of about 24 mm coil diameter. For spring material, the safe shear stress is to be limited to 420 MPa and the modulus of rigidity may be taken as 80 kN/mm^2. **[Ans. 120 mm ; 80 mm ; 3.658 mm]**

4. A multiple disc clutch has three discs on the driving shaft and two on the driven shaft, providing four pairs of contact surfaces. The outer diameter of the contact surfaces is 250 mm and the inner diameter is 150 mm. Determine the maximum axial intensity of pressure between the discs for transmitting 18.75 kW at 500 r.p.m. Assume uniform wear and coefficient of friction as 0.3.

5. A multiple disc clutch employs 3 steel and 2 bronze discs having outer diameter 300 mm and inner diameter 200 mm. For a coefficient of friction of 0.22, find the axial pressure and the power transmitted at 750 r.p.m., if the normal unit pressure is 0.13 N/mm^2.

   Also find the axial pressure of the unit normal pressure, if this clutch transmits 22 kW at 1500 r.p.m. **[Ans. 5105 N ; 44.11 kW ; 0.0324 N/mm^2]**

17-3 Cone Clutches and Brakes

a *cone clutch* shown in Fig. 17–7

- It consists of
  1. a *cup* keyed or splined to one of the shafts,
  2. a *cone* that must slide axially on splines or keys on the mating shaft, and
  3. a helical *spring* to hold the clutch in engagement.
  4. a fork that fits into the shifting groove on the friction cone for disengagement.
- The *cone angle α* between 10 and 15°.

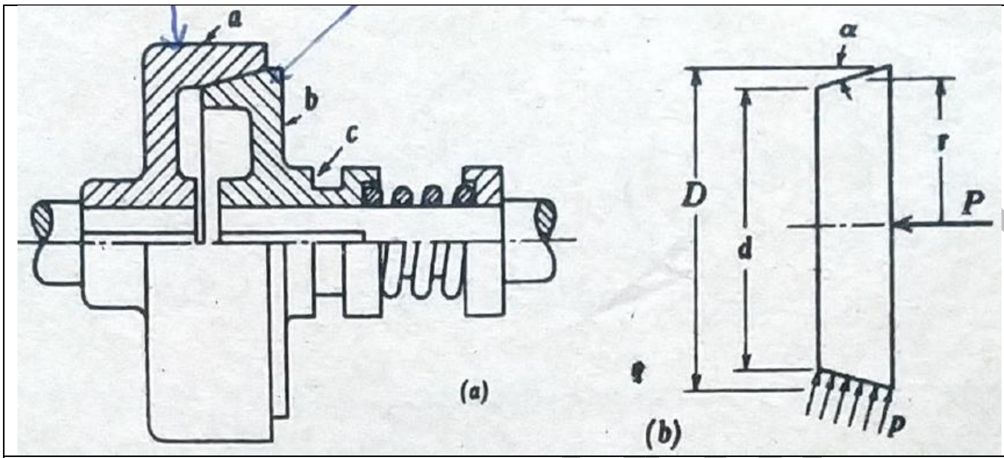


Fig. 17–7 Cone clutch.

- Force analysis The relation between the torque transmitted and the axial force may be obtained by following a similar procedure to that for the plate clutch in Art. 17-2.
- The friction surface of the inner cone of the clutch is shown in Fig. 17-7(b) and the axial force is P.

$$\text{Elementary surface area } dA = \frac{2\pi r\,dr}{\sin\alpha}$$

$$\text{Normal force on } dA = \frac{p2\pi r\,dr}{\sin\alpha}$$

$$\text{Frictional force on } dA = \frac{fp2\pi r\,dr}{\sin\alpha}$$

$$P = 2\pi \int_{d/2}^{D/2} pr\,dr \qquad (a)$$

$$T = \frac{2\pi f}{\sin\alpha} \int_{d/2}^{D/2} pr^2\,dr \qquad (b)$$

From the assumption that "the normal wear is proportional to the work of friction," as discussed in Art. 17-2, it may be deduced that

$$p = \frac{C}{r} \qquad (c)$$

By substituting the value of *p* from Eq. (*c*) into Eqs. (*a*) and (*b*), integrating, and eliminating the constant C, the following equation is determined for the torque transmitted by a cone clutch:

$$\boldsymbol{T = \frac{\mu P(D+d)}{4\sin\alpha} = \frac{\mu P r_m}{\sin\alpha}} \qquad \mathbf{(17-3)}$$

$$r_m = \frac{D+d}{4} = \frac{R+r}{2}$$

Integrating Eq. (*a*) gives the actuating force *P*

$$\boldsymbol{P = \frac{\pi p_{max} d}{2}(D - d)} \qquad \boldsymbol{(17-3b)}$$

- By making the cone angle an equal to 90°, and Eq. (3) reduces to Eq. (2).
- The torque may be transmitted by a relatively small axial force if the cone-face angle is decreased.
- There is a lower limit to the angle α, however, since the frictional force that must be overcome in releasing the clutch increases as α decreases.
- clutch with a small cone angle requires a relatively small force to engage the clutch but a large force for disengagement.
- The SAE recommends an angle α =12.5° for cone clutches faced with leather or asbestos or having cork inserts.
- Cone brakes are similar to cone clutches in construction and operation.

4 - BLOCK BRAKES AND CLUTCHES

Single-block brake In Fig. 17-8 is shown a single-block brake in which the block attached to the operating lever is forced against the rotating wheel. The frictional force produced by the block on the wheel will retard the rotation of the wheel.

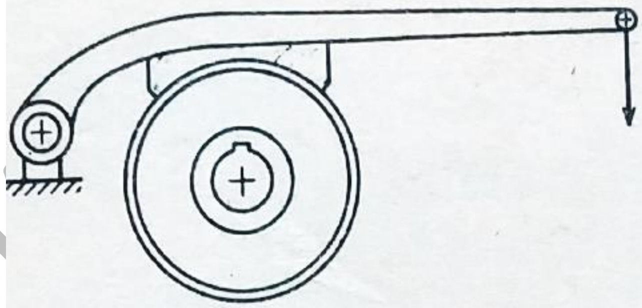


Fig. 17-8

Force analysis for friction block brake A cylindrical wheel, or drum, that is assumed to rotate as indicated is shown in Fig. 17-9.

Let P=operating force on block in radial direction

D =diameter of wheel

T = torque on wheel

θ = one-half angle of contact surface of block

b = width of wheel

μ = coefficient of friction for materials of block and wheel

p =pressure between block and wheel

Elementary arcs of contact $dA = \frac{D}{2} bd\phi$

Normal force on $dA = p\frac{D}{2} bd\phi$

The component of this normal force parallel to P is equal to

$$dP = p\frac{D}{2} b\cos\phi \, d\phi$$

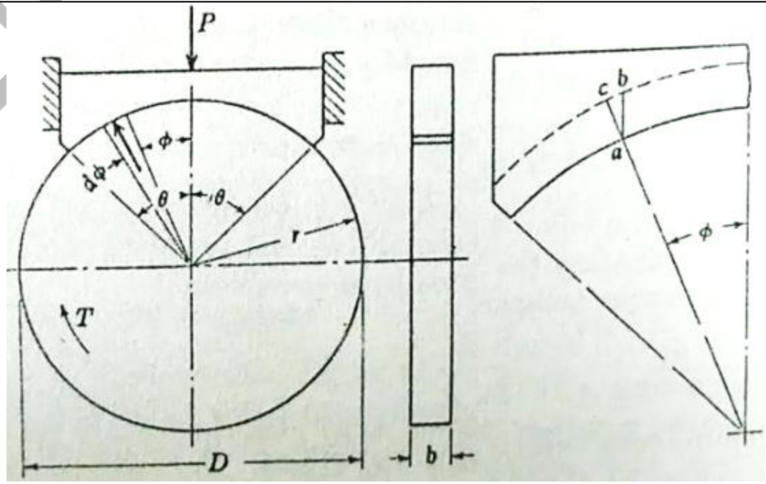


Fig. 17-9 (a) Single-block brake (b) Wear of block

$$P = \int_{-\theta}^{+\theta} p\frac{D}{2} b\cos\phi \, d\phi = \frac{Db}{2}\int_{-\theta}^{+\theta} p\cos\phi \, d\phi \qquad (a)$$

Force of friction on elementary area $= \mu p\frac{D}{2} bd\phi$

$$T = \int_{-\theta}^{+\theta} \mu p\frac{D^2}{4} bd\phi \qquad (b)$$

By assuming that the coefficient of friction is constant, the equation for T becomes

$$\boldsymbol{T = \mu \frac{D^2 b}{4} \int_{-\theta}^{+\theta} p d\phi} \qquad (\mathbf{17-4})$$

- ✓ In order to obtain an expression for the pressure in terms of ϕ, it is necessary to make the assumption "**normal wear is proportional to the work of friction**."
- ✓ From Fig. 17-9(b) the component of wear in the direction of P, that is, $\overline{ab}$, will be constant.
- ✓ The normal wear $\overline{ac} = \overline{ab}\cos\phi$
- ✓ The work of friction on an elementary area is proportional to the pressure p; hence from the assumption that the normal wear $\overline{ac}$ is proportional to the work of friction

$$p \;\alpha\; \overline{ab}\cos\phi \qquad \text{or} \qquad p = C\cos\phi$$

From Eq. (a)

$$P = \frac{CDb}{2}\int_{-\theta}^{+\theta}\cos^2\phi\, d\phi = \frac{CDb}{4}(2\theta + \sin 2\theta)$$

From Eq. (b)

$$T = \frac{C\mu D^2 b}{4}\int_{-\theta}^{+\theta}\cos\phi\, d\phi = 2C\mu b\left(\frac{D}{2}\right)^2 \sin\theta$$

By eliminating C from the two preceding equations

$$\boldsymbol{T = \mu P \frac{D}{2}\frac{4\sin\theta}{2\theta + \sin 2\theta} = \mu' P \frac{D}{2}} \qquad (\mathbf{17-5})$$

And from $F = 2T/D$;

$$\boldsymbol{F = \mu P \frac{4\sin\theta}{2\theta + \sin 2\theta}} \qquad (\mathbf{17-6})$$

Substituting the equivalent coefficient of friction $\boldsymbol{\mu' = \mu \frac{4\sin\theta}{2\theta + \sin 2\theta}}$ in eqn. (17-6)

$$\boldsymbol{F = \mu' P} \qquad (\mathbf{17-7})$$

تم رسم العلاقة بين $\frac{4\sin\theta}{2\theta+\sin 2\theta}$ ونصف زاوية التلامس $\boldsymbol{\theta}$ لغرض إيجاد معامل الاحتكاك المكافئ $\boldsymbol{\mu'}$ بسهولة كما مبين في الشكل (١٧-١٠) في أدناه

Fig. 17-10 relation between $(4\sin\theta)/(2\theta + \sin 2\theta)$ and θ

Self-Energizing in Block Brakes الكبح الذاتي في المكابح الصندوقية

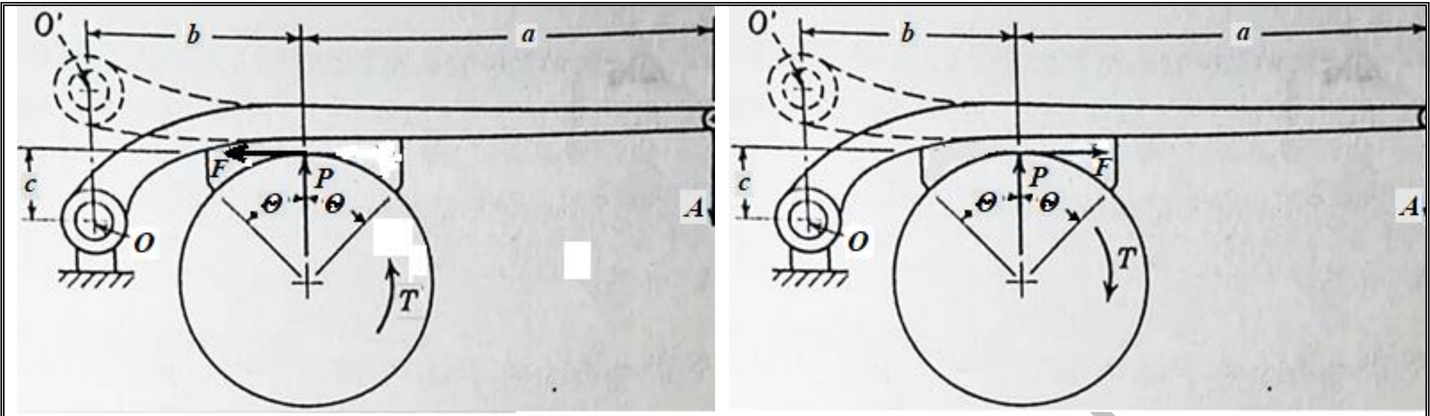


Fig. 17-11 (a) block brake lever **Fig. 17-11 (b) block brake lever**

بإجراء توازن للعزوم المؤثرة على عتلة الكابح المبين بالشكل (١٧-١١)(أ)، حول المرتكز O نحصل على:

$$[\boldsymbol{\Sigma M_O = 0}] \quad \rightarrow \quad \boldsymbol{A(a+b) - Pb - Fc = 0} \quad \boldsymbol{(17-8)}$$

وبتعويض $F = \mu' P$ من المعادلة (١٧-٧)، في المعادلة (١٧-٨) نحصل على:

$$A(a+b) - Pb - \mu' Pc = A(a+b) - P(b + \mu' c) = 0$$

ومنها نحصل على P

$$\boldsymbol{P = \frac{A(a+b)}{b + \mu' c}}$$

ثم بتعويض P في معادلة العزم (١٧-٥) نحصل على عزم الكبح للشكل (١٧-١١)(أ):

$$\boldsymbol{T = \mu' P \frac{D}{2} = \frac{\mu' AD(a+b)}{2(b + \mu' c)}} \qquad \boldsymbol{(17-9)}$$

ملاحظة١: في المعادلة (١٧-٨)، نلاحظ أن اتجاه عزم القوة F حول المرتكز O، والذي يساوي $F \times c$ ، بعكس اتجاه عزم قوة الكبح A والذي يساوي $A \times (a+b)$ وهذا يعني ان عزم القوة F يُضعف من عزم الكبح.

الكبح الذاتي:

لو عكسنا اتجاه دوران الدولاب ليكون باتجاه عقارب الساعة كما في الشكل (١٧-١١)(ب) عندئذ، يكون عزم القوة F حول المرتكز O، والذي يساوي $F \times c$ بنفس اتجاه عزم الكبح، وفي هذه الحالة إذا كانت المسافة c كافية بحيث يكون العزم $F \times c$ أكبر من او مساوٍ للعزم $P \times b$ فيمكن ان تتم عملية الكبح دون الحاجة الى قوة خارجية، أي تكون القوة اللازمة للكبح A مساوية للصفر هذه الحالة يطلق عليها الكبح الذاتي self-energizing، وهذا ما سوف نوضحه من خلال الاتي:

اذا فرضنا ان اتجاه دوران الدولاب drum باتجاه عقارب الساعة كما في الشكل (١٧-١١)(ب) ، فان المعادلة (١٧-٨) تصبح

$$[\boldsymbol{\Sigma M_O = 0}] \quad \rightarrow \quad \boldsymbol{A(a+b) - Pb + Fc = 0} \quad \boldsymbol{(17-8)b}$$

وبجعل القوة اللازمة للكبح في المعادلة $\boldsymbol{(17-8)b}$ مساوية او اقل من صفر $A \le 0$ وهي حالة الكبح الذاتي self-energizing فأن

$$\boldsymbol{-P \times b + F \times c \ge 0}$$

او بعبارة أخرى

$$\boldsymbol{F \times c \ge P \times b}$$

إذا تحقق هذا الشرط وذلك بأن يكون $F \times c \geq P \times b$ فهذا يعني ان عزم قوة الاحتكاك $F \times c$ يكون اكبر من عزم القوة العمودية $P \times b$ وبالتالي فان الدولاب يتوقف عن الحركة دون الحاجة الى قوة خارجية A هذه الحالة يطلق عليها الكبح الذاتي self-energizing وشرطها أن يكون $F \times c \geq P \times b$ والان لو عوضنا عن F من المعادلة (٧-١٧) بـ

$$F = \mu' P$$

فهذا يعني ان

$$\mu' P \times c \geq P \times b$$

وبحذف P من الطرفين نحصل على

$$\mu' \times c \geq b \qquad or \qquad c \geq b/\mu'$$

بهذا الشرط تحقق ظاهرة الكبح الذاتي وهو ان يكون $c \geq b/\mu'$ وهذه تكون مفيدة في بعض الحالات لمنع الدوران العكسي عند فشل المنظومة او التوقف المفاجئ للماكنة

Example 25.2. *A single block brake is shown in Fig. 25.5. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.*

Solution. Given : d = 250 mm or r = 125 mm ; $2\theta = 90° = \pi / 2$ rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60° , therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu' . R_N$

Taking moments above the fulcrum O, we have

$$700\,(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\ F_t$$

or $520\ F_t - 50\ F_t = 700 \times 450$ or $F_t = 700 \times 450 / 470 = 670$ N

We know that torque transmitted by the block brake,

$$T_B = F_t \times r = 670 \times 125 = 83\ 750 \text{ N-mm} = 83.75 \text{ N-m}$$ **Ans.**

Example 25.3. *Fig. 25.6 shows a brake shoe applied to a drum by a lever AB which is pivoted at a fixed point A and rigidly fixed to the shoe. The radius of the drum is 160 mm. The coefficient of friction of the brake lining is 0.3. If the drum rotates clockwise, find the braking torque due to the horizontal force of 600 N applied at B.*

Solution. Given : r = 160 mm = 0.16 m ; μ = 0.3 ; P = 600 N

Since the angle subtended by the shoe at the centre of the drum is 40°, therefore we need not to calculate the equivalent coefficient of friction (μ').

Let R_N = Normal force pressing the shoe on the drum, and

F_t = Tangential braking force = $\mu . R_N$

Taking moments about point A,

$$R_N \times 350 + F_t (200 - 160) = 600 (400 + 350)$$

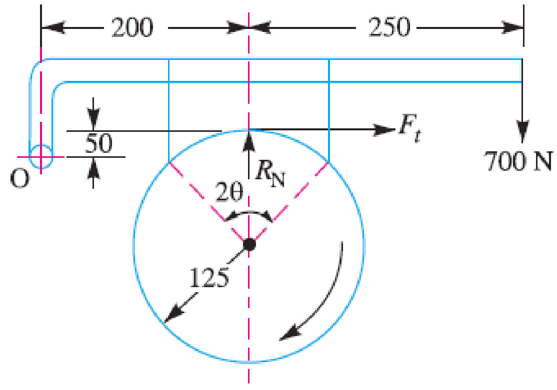
$$\frac{F_t}{0.3} \times 350 + 40\, F_t = 600 \times 750$$

or $$207\, F_t = 450 \times 10^3$$

$$\therefore \quad F_t = 450 \times 10^3 / 1207 = 372.8 \text{ N}$$

We know that braking torque,

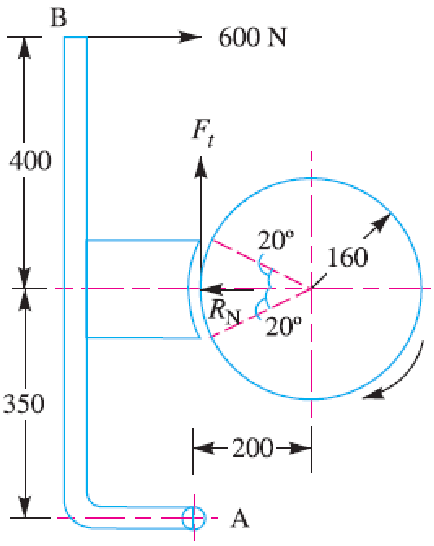
$$T_B = F_t \times r = 372.8 \times 0.16$$
$$= 59.65 \text{ N-m} \quad \textbf{Ans.}$$

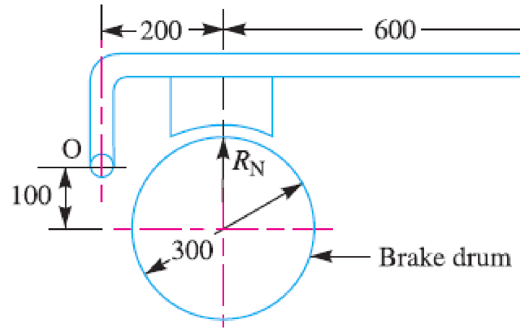


All dimensions in mm.

Fig. 25.5

Brakes on a car wheel (inner side)





All dimensions in mm.

Fig. 25.6 **Fig. 25.7**

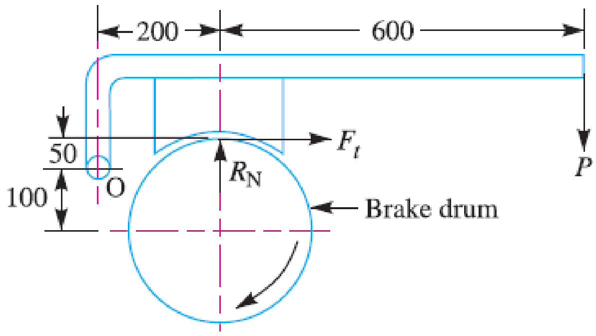
Example 25.4. *The block brake, as shown in Fig. 25.7, provides a braking torque of 360 N-m. The diameter of the brake drum is 300 mm. The coefficient of friction is 0.3. Find :*

1. *The force (P) to be applied at the end of the lever for the clockwise and counter clockwise rotation of the brake drum; and*
2. *The location of the pivot or fulcrum to make the brake self locking for the clockwise rotation of the brake drum.*

Solution. Given : $T_B = 360$ N-m $= 360 \times 10^3$ N-mm ; $d = 300$ mm or $r = 150$ mm $= 0.15$ m ; $\mu = 0.3$

1. *Force (P) for the clockwise and counter clockwise rotation of the brake drum*

For the clockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.8.



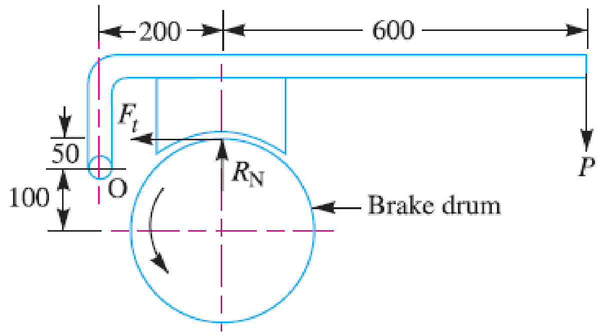


Fig. 25.8 **Fig. 25.9**

We know that braking torque (T_B),

$$360 = F_t \times r = F_t \times 0.15 \quad \text{or} \quad F_t = 360 / 0.15 = 2400 \text{ N}$$

and normal force, $R_N = F_t / \mu = 2400 / 0.3 = 8000$ N

Now taking moments about the fulcrum O, we have

$$P(600 + 200) + F_t \times 50 = R_N \times 200$$
$$P \times 800 + 2400 \times 50 = 8000 \times 200$$
$$P \times 800 = 8000 \times 200 - 2400 \times 50 = 1480 \times 10^3$$
$$\therefore \quad P = 1480 \times 10^3 / 800 = 1850 \text{ N} \quad \textbf{Ans.}$$

For the counter clockwise rotation of the drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.9.

Taking moments about the fulcrum O, we have

$$P(600 + 200) = F_t \times 50 + R_N \times 200$$
$$P \times 800 = 2400 \times 50 + 8000 \times 200 = 1720 \times 10^3$$
$$\therefore \quad P = 1720 \times 10^3 / 800 = 2150 \text{ N} \quad \textbf{Ans.}$$

2. Location of the pivot or fulcrum to make the brake self-locking

The clockwise rotation of the brake drum is shown in Fig. 25.8. Let x be the distance of the pivot or fulcrum O from the line of action of the tangential force (F_t). Taking moments about the fulcrum O, we have

$$P(600 + 200) + F_t \times x - R_N \times 200 = 0$$

In order to make the brake self-locking, $F_t \times x$ must be equal to $R_N \times 200$ so that the force P is zero.

$$\therefore \quad F_t \times x = R_N \times 200$$
$$2400 \times x = 8000 \times 200 \quad \text{or} \quad x = 8000 \times 200 / 2400 = 667 \text{ mm} \quad \textbf{Ans.}$$

5 – DOUBLE BLOCK BRAKES AND CLUTCHES المكابح والفواصل ثنائية البطانة

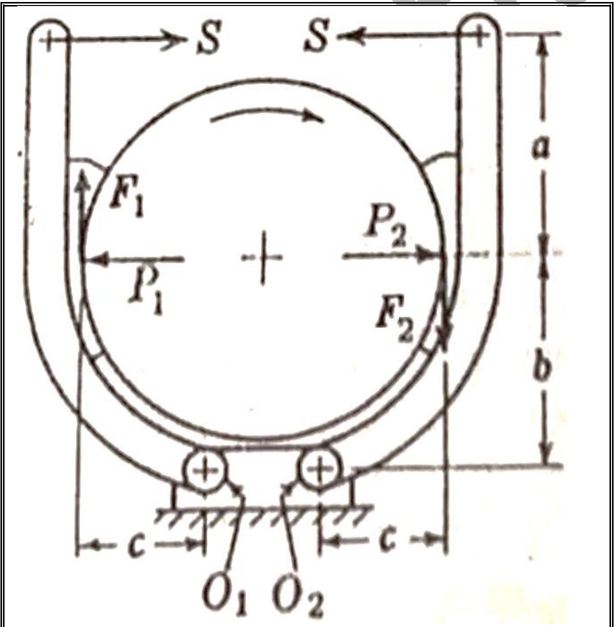


Fig. 17-13 Double - block brake

الهدف من التحليل:

الشكل (١٧-١٣) يبين كابحا مزدوج البطانة. الهدف هو ايجاد قوة النابض S اللازمة للكبح إذا كان عزم مقداره T مسلط على دولاب Drum قطره D،

١- **الخطوة الأولى:** إيجاد محصلة العزوم حول المركزين O1 و O2 من معادلات التوازن السكوني

$$\Sigma M_{o_1} = S(a + b) + F_1 c - P_1 b = 0$$
$$\Sigma M_{o_2} = S(a + b) - F_2 c - P_2 b = 0$$

٢- العلاقة بين القوة F والقوة P يمكن ايجادها من المعادلة (١٧-٧):

$$\frac{F_1}{P_1} = \frac{F_2}{P_2} = \mu'$$

٣- من العلاقات في اعلاه يمكن حساب القوتين F_1 و F_2 بدلالة قوة النابض S

$$F_1 = \frac{(a + b)\mu'}{b - \mu' c} S$$
$$F_2 = \frac{(a + b)\mu'}{b + \mu' c} S$$

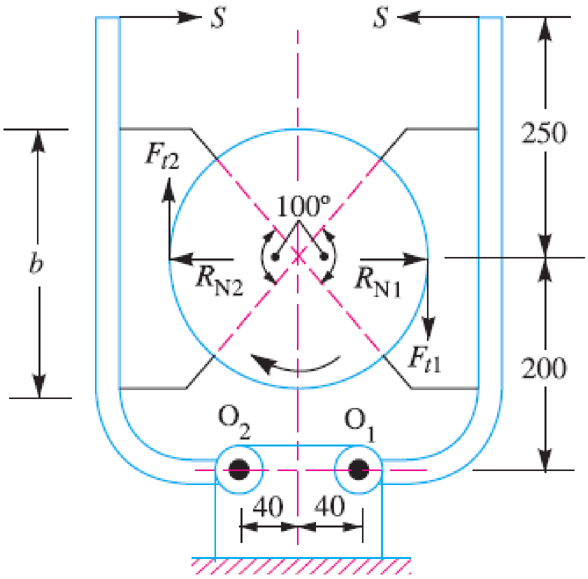
٤- حساب عزم الكبح من المعادلة الاتية:

$$T = (F_1 + F_2) \times \frac{D}{2} \qquad (17-10)$$

ملاحظة١: بسبب الفرق بين قيمتي القوتين F_1 و F_2 فان البلى للبطانتين يختلف اي غير متساوي للجانبين، إذا كان الدولاب يدور باتجاه واحد. بما ان الحشوات تكون عادة عالية الجودة واغلب المكابح تعمل بالاتجاهين هذا يؤدي الى مساواة البلى للجانبين.

ملاحظة٢: إذا كان المفصلان O_1 و O_2، في الشكل (١٧-١٣) يقعان على امتداد القوتين F_1 و F_2 فان البطانتين لهما نفس البلى الا ان تأثير وضع المفصل بالطريقة المبينة بالشكل (١٧-١٣) يؤدي الى تقليل قوة النابض اللازمة لمقاومة العزم الى اقل منها فيما لو كان المفصلان على امتداد القوتين F_1 و F_2.

Example 25.6. *A double shoe brake, as shown in Fig. 25.11 is capable of absorbing a torque of 1400 N-m. The diameter of the brake drum is 350 mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4; find : 1. the spring force necessary to set the brake; and 2. the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.3 N/mm².*



All dimensions in mm.

Fig. 25.11

Solution. Given : T_B = 1400 N-m = 1400 × 10³ N-mm ; d = 350 mm or r = 175 mm ; 2θ = 100° = 100 × π / 180 = 1.75 rad ; μ = 0.4 ; p_b = 0.3 N/mm²

1. ***Spring force necessary to set the brake***

Let S = Spring force necessary to set the brake,

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe, and

R_{N2} and F_{t2} = Corresponding values on the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 50°}{1.75 + \sin 100°} = 0.45$$

Taking moments about the fulcrum O_1, we have

$$S \times 450 = R_{N1} \times 200 + F_{t1}\,(175 - 40) = \frac{F_{t1}}{0.45} \times 200 + F_{t1} \times 135 = 579.4\,F_{t1}$$

...(Substituting $R_{N1} = F_{t1}/\mu'$)

$$\therefore \quad F_{t1} = S \times 450 / 579.4 = 0.776\,S$$

Again taking moments about O_2, we have

$$S \times 450 + F_{t2}(175 - 40) = R_{N2} \times 200 = \frac{F_{t2}}{0.45} \times 200 = 444.4\ F_{t2}$$

...(Substituting $R_{N2} = F_{t2}/\mu'$)

$$444.4\ F_{t2} - 135\ F_{t2} = S \times 450 \quad \text{or} \quad 309.4\ F_{t2} = S \times 450$$

$$\therefore \quad F_{t2} = S \times 450 / 309.4 = 1.454\ S$$

We know that torque capacity of the brake (T_B),

$$1400 \times 10^3 = (F_{t1} + F_{t2})\ r = (0.776\ S + 1.454\ S)\ 175 = 390.25\ S$$

$$\therefore \quad S = 1400 \times 10^3 / 390.25 = 3587 \text{ N}$$ **Ans.**

2. Width of the brake shoes

Let b = Width of the brake shoes in mm.

We know that projected bearing area for one shoe,

$$A_b = b\,(2r \sin\theta) = b\,(2 \times 175 \sin 50°) = 268\ b \text{ mm}^2$$

∴ Normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.776 \times S}{0.45} = \frac{0.776 \times 3587}{0.45} = 6186 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{1.454 \times S}{0.45} = \frac{1.454 \times 3587}{0.45} = 11\,590 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that the bearing pressure on the lining material (p_b),

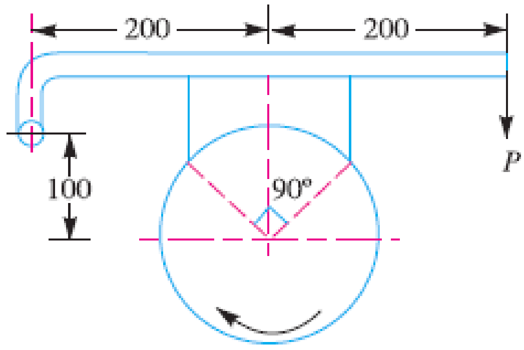
$$0.3 = \frac{R_{N2}}{A_b} = \frac{11\,590}{268b} = \frac{43.25}{b}$$

$$\therefore \quad b = 43.25 / 0.3 = 144.2 \text{ mm}$$ **Ans.**

الواجب البيتي

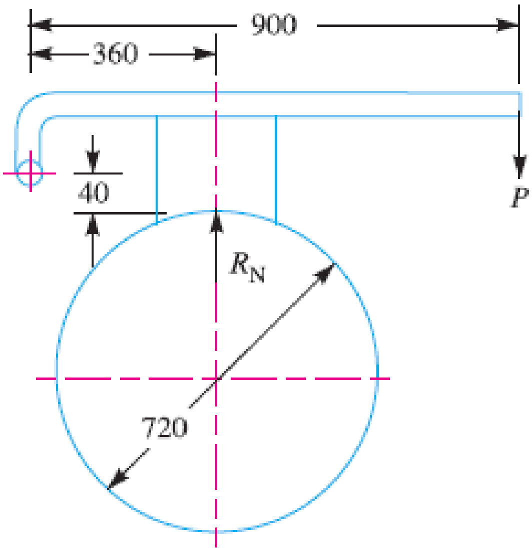
2. A single block brake, as shown in Fig. 25.38, has the drum diameter 250 mm. The angle of contact is 90° and the coefficient of friction between the drum and the lining is 0.35. If the torque transmitted by the brake is 70 N-m, find the force *P* required to operate the brake.

[Ans. 700 N]



All dimensions in mm.

Fig. 25.38



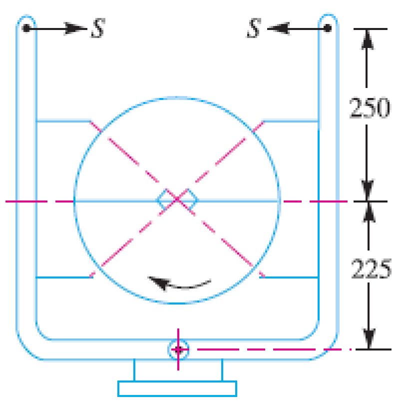
All dimensions in mm.

Fig. 25.39

3. A single block brake, as shown in Fig. 25.39, has a drum diameter of 720 mm. If the brake sustains 225 N-m torque at 500 r.p.m.; find :
   (*a*) the required force (*P*) to apply the brake for clockwise rotation of the drum;
   (*b*) the required force (*P*) to apply the brake for counter clockwise rotation of the drum;
   (*c*) the location of the fulcrum to make the brake self-locking for clockwise rotation of the drum; and

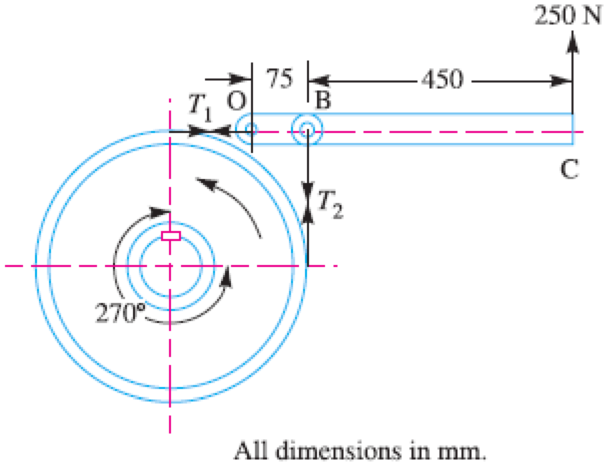
   The coefficient of friction may be taken as 0.3. **[Ans. 805.4 N ; 861 N; 1.2 m ; 11.78 kW]**

4. The layout and dimensions of a double shoe brake is shown in Fig. 25.40. The diameter of the brake drum is 300 mm and the contact angle for each shoe is 90°. If the coefficient of friction for the brake lining and the drum is 0.4, find the spring force necessary to transmit a torque of 30 N-m. Also determine the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.28 N/mm^2. **[Ans. 99.1 N ; 5 mm]**



All dimensions in mm.

Fig. 25.40



All dimensions in mm.

Fig. 25.41

6 – BAND BRAKES المكابح الطوقية

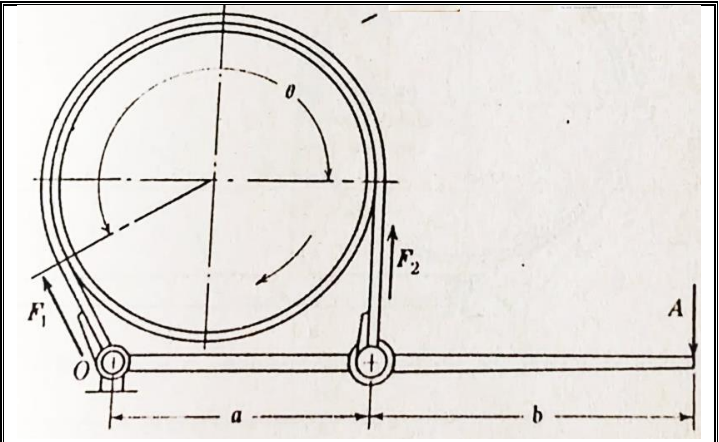


Fig. 17-15 Band brake

الحزام الطوقي مبين بالشكل (١٧-١٥) يتم شد الطوق بواسطة القوة A عن طريق العتلة OA،
يتم صناعة الطوق من الجلد او القنب convas المغطس بالمطاط او من شرائط الفولاذ المغطاة بالخشب
لحاله الدوران باتجاه عقارب الساعة للدولاب كما مبين بالشكل (١٧-١٥) فان الشد F_1 ستكون أكبر من F_2

$$\Sigma M_o = F_1 O + F_2 a - A(a+b) = 0 \;.....(a)$$

يمكن إيجاد العلاقة بين القوتين F_1 و F_2 من العلاقة الخاصة بالأحزمة بعد حذف مركبة الطرد المركزي F_c

$$\frac{F_1}{F_2} = e^{\mu\theta} \;.......(b)$$

وكذلك العزم على دولاب يمكن ايجاده من العلاقة الاتية:

$$T = (F_1 - F_2)\frac{D}{2} \quad ......(c)$$

من العلاقات (a) و (b) و (c) في اعلاه يمكن ايجاد علاقة تربط بيت القوة A، المسلطة على العتلة وبين العزم T على الدولاب وكما يأتي:

$$A = \frac{2Ta}{D(a+b)(e^{\mu\theta}-1)} \quad ....(17-11)$$

للمكابح الطوقية استخدامات واسعة في الصناعة لبساطتها وموثوقيتها، كما انها مناسبة للاستخدام من المكائن كبيرة الحجم.

Example 25.9. *A band brake acts on the $\frac{3}{4}$th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the operating force when the drum rotates in the anticlockwise direction.*

If the brake lever and pins are to be made of mild steel having permissible stresses for tension and crushing as 70 MPa and for shear 56 MPa, design the shaft, key, lever and pins. The bearing pressure between the pin and the lever may be taken as 8 N/mm².

Solution. Given : $d = 450$ mm or $r = 225$ mm ; $T_B = 225$ N-m $= 225 \times 10^3$ N-mm ; $OB = 100$ mm ; $l = 500$ mm ; $\mu = 0.25$; $\sigma_t = \sigma_c = 70$ MPa $= 70$ N/mm^2 ; $\tau = 56$ MPa $= 56$ N/mm^2 ; $p_b = 8$ N/mm^2

Operating force

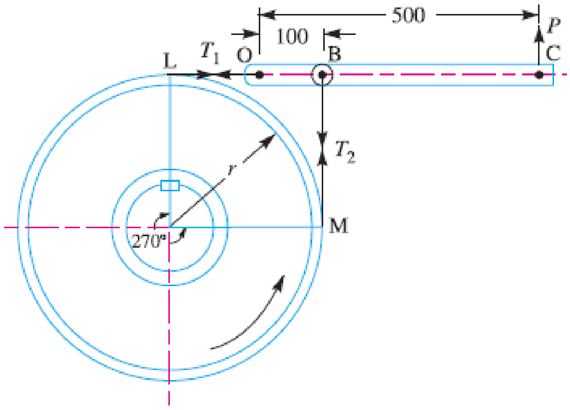
Let P = Operating force.

The band brake is shown in Fig. 25.17. Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2.

We know that angle of wrap,

$$\theta = \frac{3}{4}\text{th of circumference} = \frac{3}{4} \times 360° = 270°$$

$$= 270 \times \frac{\pi}{180} = 4.713 \text{ rad}$$



All dimensions in mm.

and $$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5123 \quad \text{or} \quad \frac{T_1}{T_2} = 3.25 \quad ...(i)$$

...(Taking antilog of 0.5123)

We know that braking torque (T_B),

$$225 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 225$$

$$\therefore \quad T_1 - T_2 = 225 \times 10^3 / 225 = 1000 \text{ N} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$T_1 = 1444 \text{ N and } T_2 = 444 \text{ N}$$

Taking moments about the fulcrum *O*, we have

$$P \times 500 = T_2 \times 100 = 444 \times 100 = 44\,400$$

$$\therefore \quad P = 44\,400 / 500 = 88.8 \text{ N} \quad \textbf{Ans.}$$

Drums for band brakes.

Design of shaft

Let d_s = Diameter of the shaft in mm.

Since the shaft has to transmit torque equal to the braking torque (T_B), therefore

$$225 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_s)^3 = \frac{\pi}{16} \times 56\, (d_s)^3 = 11\, (d_s)^3$$

$$\therefore \quad (d_s)^3 = 225 \times 10^3/11 = 20.45 \times 10^3 \text{ or } d_s = 27.3 \text{ say } 30 \text{ mm} \quad \textbf{Ans.}$$

Design of key

The standard dimensions of the key for a 30 mm diameter shaft are as follows :

Width of key, w = 10 mm **Ans.**

Thickness of key, t = 8 mm **Ans.**

Let l = Length of key.

Considering the key in shearing, we have braking torque (T_B),

$$225 \times 10^3 = l \times w \times \tau \times \frac{d_s}{2} = l \times 10 \times 56 \times \frac{30}{2} = 8400\, l$$

$$\therefore \quad l = 225 \times 10^3 / 8400 = 27 \text{ mm}$$

Now considering the key in crushing, we have braking torque (T_B),

$$225 \times 10^3 = l \times \frac{t}{2} \times \sigma_c \times \frac{d_s}{2} = l \times \frac{8}{2} \times 70 \times \frac{30}{2} = 4200\, l$$

$\therefore$ $l = 225 \times 10^3 / 4200 = 54$ mm

Taking larger of two values, we have $l = 54$ mm **Ans.**

Design of lever

Let t_1 = Thickness of the lever in mm, and

B = Width of the lever in mm.

The lever is considered as a cantilever supported at the fulcrum O. The effect of T_2 on the lever for determining the bending moment on the lever is neglected. This error is on the safer side.

$\therefore$ Maximum bending moment at O due to the force P,

$$M = P \times l = 88.8 \times 500 = 44\ 400 \text{ N-m}$$

Section modulus,

$$Z = \frac{1}{6} t_1 . B^2 = \frac{1}{6} t_1 (2t_1)^2 = 0.67 (t_1)^3 \text{ mm}^3 \quad \text{...(Assuming } B = 2t_1)$$

We know that the bending stress (σ_t),

$$70 = \frac{M}{Z} = \frac{44\ 400}{0.67 (t_1)^3} = \frac{66\ 300}{(t_1)^3}$$

$\therefore$ $(t_1)^3 = 66\ 300 / 70 = 947$ or $t_1 = 9.82$ say 10 mm **Ans.**

and $B = 2\, t_1 = 2 \times 10 = 20$ mm **Ans.**

Design of pins

Let d_1 = Diameter of the pins at O and B, and

l_1 = Length of the pins at O and $B = 1.25\, d_1$ ...(Assume)

The pins at O and B are designed for the maximum tension in the band (*i.e.* $T_1 = 1444$ N),

Considering bearing of the pins at O and B, we have maximum tension (T_1),

$$1444 = d_1 . l_1 . p_b = d_1 \times 1.25\, d_1 \times 8 = 10\, (d_1)^2$$

$\therefore$ $(d_1)^2 = 1444 / 10 = 144.4$ or $d_1 = 12$ mm **Ans.**

and $l_1 = 1.25\, d_1 = 1.25 \times 12 = 15$ mm **Ans.**

Let us now check the pin for induced shearing stress. Since the pin is in double shear, therefore maximum tension (T_1),

$$1444 = 2 \times \frac{\pi}{4} (d_1)^2\, \tau = 2 \times \frac{\pi}{4} (12)^2\, \tau = 226\, \tau$$

$\therefore$ $\tau = 1444 / 226 = 6.4 \text{ N/mm}^2 = 6.4$ MPa

This induced stress is quite within permissible limits.

The pin may be checked for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W . l_1 = \frac{5}{24} \times 1444 \times 15 = 4513 \text{ N-mm}$$

... (Here $W = T_1 = 1444$ N)

and section modulus, $Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(12)^3 = 170 \text{ mm}^3$

$\therefore$ Bending stress induced

$$= \frac{M}{Z} = \frac{4513}{170} = 26.5 \text{ N-mm}^2 = 26.5 \text{ MPa}$$

This induced bending stress is within safe limits of 70 MPa.

The lever has an eye hole for the pin and connectors at band have forked end.

Thickness of each eye,

$$t_2 = \frac{l_1}{2} = \frac{15}{2} = 7.5 \text{ mm}$$

Outer diameter of the eye,

$$D = 2d_1 = 2 \times 12 = 24 \text{ mm}$$

A clearance of 1.5 mm is provided on either side of the lever in the fork.

A brass bush of 3 mm thickness may be provided in the eye of the lever.

$\therefore$ Diameter of hole in the lever

$$= d_1 + 2 \times 3 = 12 + 6 = 18 \text{ mm}$$

الواجب البيتي

5. The drum of a simple band brake is 450 mm. The band embraces 3/4th of the circumference of the drum. One end of the band is attached to the fulcrum pin and the other end is attached to a pin *B* as shown in Fig. 25.41. The band is to be lined with asbestos fabric having a coefficient of friction 0.3. The allowable bearing pressure for the brake lining is 0.21 N/mm². Design the band shaft, key, lever and fulcrum pin. The material of these parts is mild steel having permissible stresses as follows :

$$\sigma_t = \sigma_c = 70 \text{ MPa, and } \tau = 56 \text{ MPa}$$

7– DIFFERENTIAL BAND BRAKES المكابح الطوقية التفاضلية

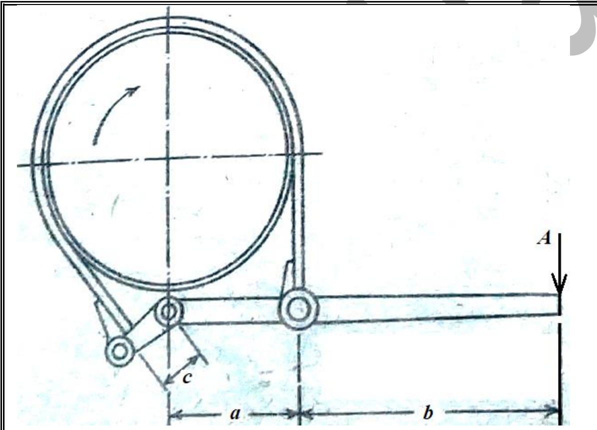


Fig. 17-15 Band brake

في هذا النوع من المكابح، تساعد مركبة الشد F_1 على تسليط قوة كبح (كبح ذاتي) كما في الشكل (١٧-١٧)، اذ نحصل من معادلة الاتزان على:

$$A(a+b) - aF_2 + cF_1 = 0$$

هذه المعادلة يمكن اعادة ترتيبها بالشكل الآتي:

$$\boldsymbol{A = \frac{F_1}{a+b}\left(\frac{a}{e^{\mu\theta}} - c\right)}$$

في هذه المعادلة اذا كان الحد $a/e^{\mu\theta}$ اقل من c يكون الطرف الايمن من المعادلة سالبا، في هذه الحالة يقال ان المكبح فيه اقفال ذاتي للدوران باتجاه عقرب الساعة، وهذه الميزة غير مرغوب فيها في المكابح المستخدمة للسيطرة على السرعة، لكنها تكون مفيدة في منع الدوران العكسي للدولاب اذ انها، تمنع دوران الدولاب بعكس اتجاه دوران المنظومة، وهذا مفيد في بعض التطبيقات كالأحزمة الناقلة conveyer التي تستخدم على اسطح مائلة، لمنع الحركة العكسية للدولاب عند التوقف الطارئ للمنظومة او انقطاع الكهرباء عندما يكون الحزام الناقل محملا.

ملاحظة: يفضل تصميم المكابح بحيث تثبت على محور عمود المحرك وذلك لان العزم فيه يكون الاقل في المنظومة، وهذا يتطلب عزما اصغر مما لو تم تثبيته على الاعمدة ذات السرع الواطئة.

ملاحظات مهمة:

- الضغط المسموح به على بطانة الكابح الطوقي $\left(p_a = \frac{2F_1}{bD}\right)$

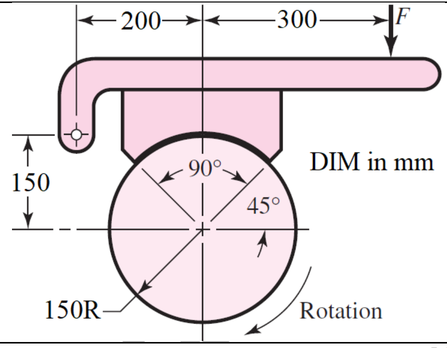
F_1: اكبر قوة شد

b: عرض بطانة الطوق

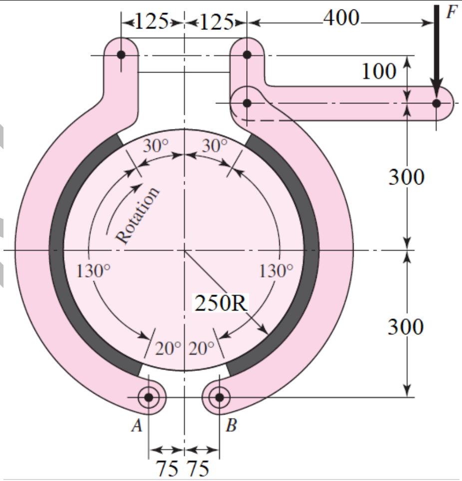
D: قطر الدولاب

PROBLEMS

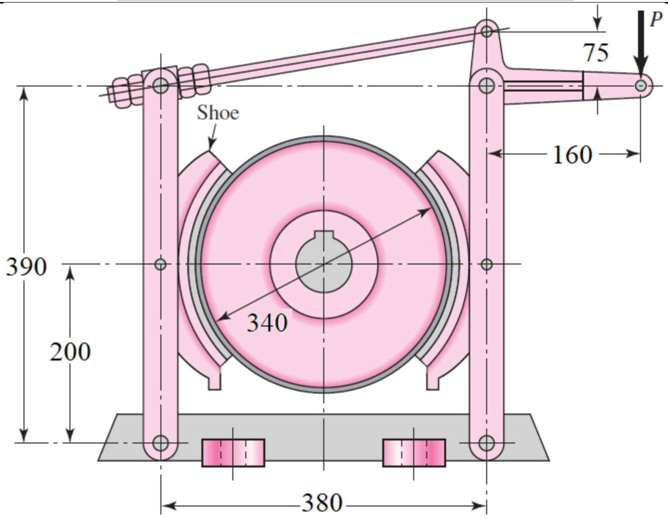
Problem 1 (Dimensions in mm.)
The block-type hand brake shown in the figure has a face width of 30 mm and a mean coefficient of friction of 0.25. For an estimated actuating force of 400 N, find the maximum pressure on the shoe and find the braking torque.



Problem 2 (Dimensions in mm.)
The brake shown in the figure has a coefficient of friction of 0.30, a face width of 75 mm, and a limiting shoe lining pressure $pV = 1.7\ \text{MPa} \times \text{m/s}$. Find the limiting actuating force *F* and the torque capacity.

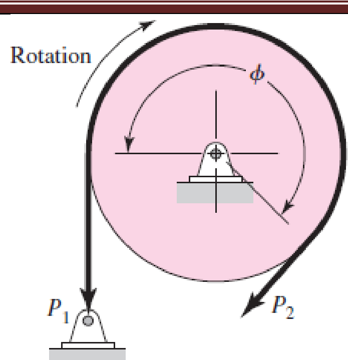


Problem 3 (Dimensions in mm.)
The shoes on the brake depicted in the figure subtend a 90◦ arc on the drum of this external pivoted-shoe brake. The actuation force *P* is applied to the lever. The rotation direction of the drum is counterclockwise, and the coefficient of friction is 0.30.
(*a*) Draw the free-body diagrams of the handle lever and both shoe levers, with forces expressed in terms of the actuation force *P*.
(b) Does the direction of rotation of the drum affect the braking torque?



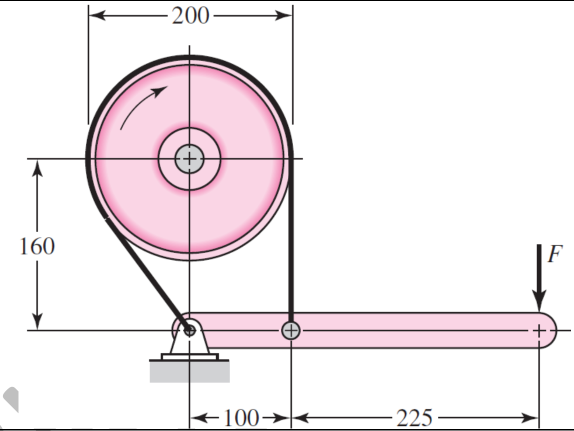
Problem 4

The maximum band interface pressure on the brake shown in the figure is 620 kPa. Use a 350 mm diameter drum, a band width of 25 mm, a coefficient of friction of 0.30, and an angle-of-wrap of 270◦. Find the band tensions and the torque capacity.



Problem 5

The brake shown in the figure has a coefficient of friction of 0.30 and is to operate using a maximum force *F* of 400 N. If the band width is 50 mm, find the band tensions and the braking torque.

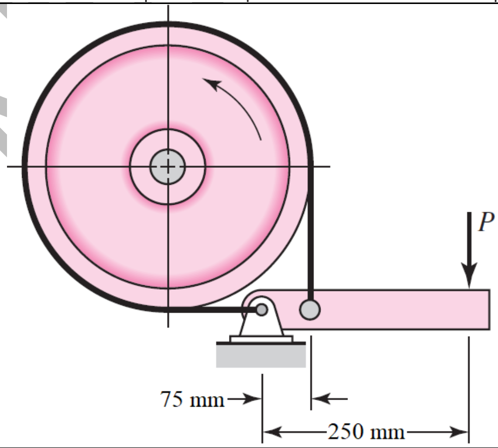


Problem 6

The figure depicts a band brake whose drum rotates counterclockwise at 200 rev/min. The drum diameter is 16 in and the band lining 75 mm wide. The coefficient of friction is 0.20. The maximum lining interface pressure is pV=1.5 MPa.m/s.

(*a*) Find the brake torque, necessary force *P*, and steady-state power.

(*b*) Complete the free-body diagram of the drum. Find the bearing radial load that a pair of straddle-mounted bearings would have to carry.



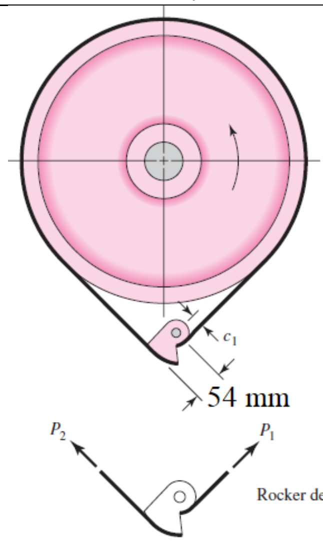
Problem 7

The figure shows a band brake designed to prevent “backward” rotation of the shaft. The angle of wrap is 270◦, the band width is 54 mm, and the coefficient of friction is 0.20. The torque to be resisted by the brake is 200 N.m. The diameter of the pulley is 210 mm

(*a*) What dimension $c1$ will just prevent backward motion?

(*b*) If the rocker was designed with $c1$ = 25 mm, what is the maximum pressure between the band and drum at 200 Nm back torque?

(*c*) If the back-torque demand is 136 Nm, what is the largest pressure between the band and drum?



Problem 8

A plate clutch has a single pair of mating friction surfaces 250-mm OD by 175-mm ID. The mean value of the coefficient of friction is 0.30, and the actuating force is 4 kN.

(*a*) Find the maximum pressure and the torque capacity using the uniform-wear model.

(*b*) Find the maximum pressure and the torque capacity using the uniform-pressure model.

Problem 9

A hydraulically operated multidisk plate clutch has an effective disk outer diameter of 165mm and an inner diameter of 100 mm. The coefficient of friction is 0.24, and the limiting pressure is 830 kPa. There are six planes of sliding present.

(*a*) Using the uniform wear model, estimate the axial force *F* and the torque *T*.

(*b*) Let the inner diameter of the friction pairs *d* be a variable. Complete the following table:

d, mm	50	75	100	125	150
T, N.m					

Problem 10

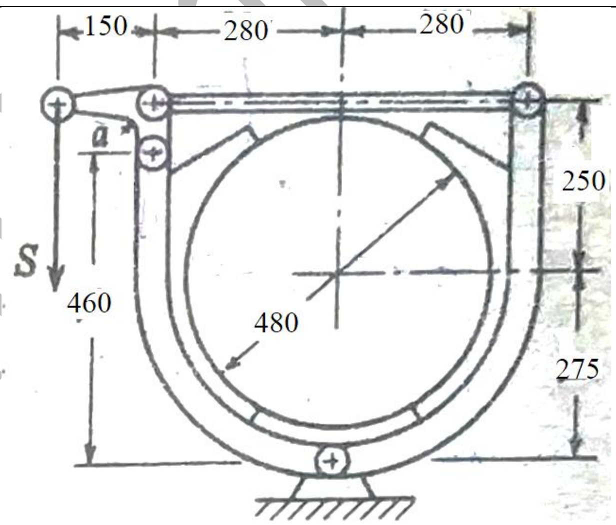
A cone clutch has *D* = 300 mm, *d* = 275 mm, a cone length of 57 mm, and a coefficient of friction of 0.28. A torque of 200 N.m is to be transmitted. For this requirement, estimate the actuating force and pressure.

Problem 11

For a band brake similar to that shown in Fig 17-15, determine the torque in N.m on the wheel for a force A on the operating lever equal to 90 N (assume that the wheel is 230 mm in diameter, a =200 mm, b=250 mm., angle of contact=210°, and the coefficient of friction =0.2 and

Problem 12

the Figure shows the arrangement and dimensions for the preliminary design of double-block brake to be set by a spring providing a force S on the bell crank *a*. the brake is to have a torque rating of 3250 N.m at 250 rpm. Assuming that the angle of contact for each shoe is 120° and that the coefficient of friction for the materials in contact is 0.35, determine the following: (a) the direction of rotation that requires the largest spring force for the rated torque and the value of the spring force S for that direction; (6) the width of shoes required, assuming $pV = 1.4\ \text{MPa} \cdot \text{m/s}$ of projected area; (c) the ratio of shoe width to drum diameter. Is this a satisfactory ratio?



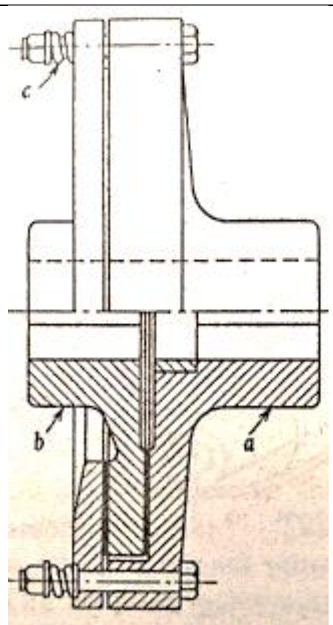
Problem 13

A slip coupling, as shown in the figure, is designed to slip when the load on the driven member becomes excessive the pressure on both faces of the flange, which is part of the hub *a*, is produced by a number of springs, such as the one shown at c. Using the data given below, determine the deflection with which each spring must be set up in order that the coupling will slip at a torque corresponding to 20 percent overload on a 19-kW motor running at 300 rpm. Use the equation based on the assumption that the normal wear is proportional to the work of friction.

Outer diameter of friction disks 790 mm

Inner diameter of friction disks 480 mm

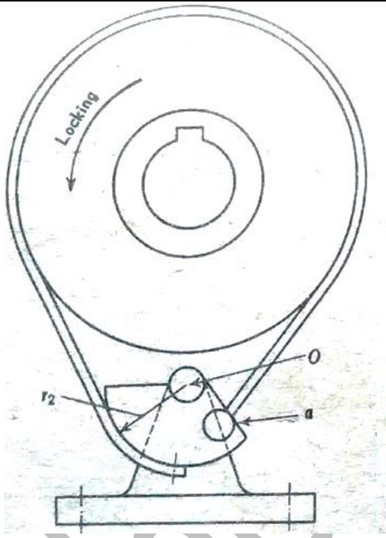
Number of bolts and springs 12



Coefficient of friction (assumed) 0.14
Spring rate for each spring 440 kN/m

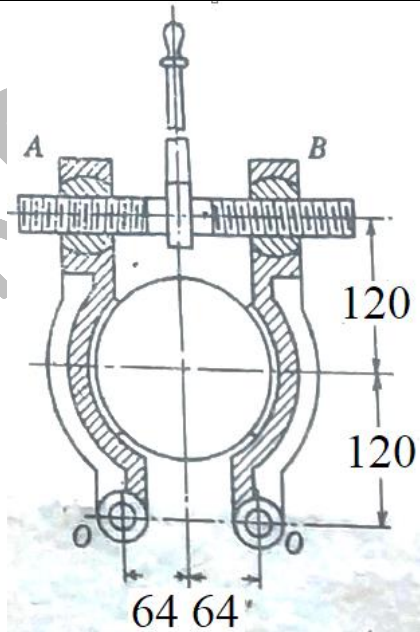
Problem 14

The backstop shown schematically in the figure is used to prevent backward rotation of the shaft. A sector is pivoted at 0, and one end of the band is attached to it and operates at a radius r2 =57 mm. The other end of the bend is attached at point *a* so that Oa =r1= 25 mm. The diameter of the wheel is 210 mm, the angle of Wrap is 270^{o}, and the width of the band is 54 mm. The torque on the wheel is 400 N.m. Assuming a coefficient of friction between the band and wheel equal to 0.2, determine the following:
(a) the maximum band tension; (6) the maximum pressure between the band and wheel; and (c) whether the backstop is self-locking.



Problem 14 (DIM in mm)

The arrangement of a transmission brake is shown in Figure. The arms are pivoted at O and, when force is applied at the end of the hand lever, the screw AB will rotate. The left- and right-hand threads working in nuts on the ends of the arms will move the arms together and thus apply the brake The force on the hand lever is applied 380 mm from the axis of the screw. The drum is 175 mm in diameter, and the angle subtended by each block is 90^{o} The screw has six square threads with a mean diameter of 20 mm and a lead of 54 mm. Assuming a coefficient of friction for the braking surfaces that is equal to 0.30 and for the threads equal to 0.15, determine the force on the hand lever required to set the brake when the torque on the drum is 200 Nm.



Problem 15 (DIM in mm)

The wheel of the double-block spring-set brake shown in Fig is 300 mm in diameter and the dimensions are a =200 mm. and b = 250 mm. Assuming that there is a coefficient of friction of 0.4 and that the angle of contact for each block is 110°, determine the spring force that is required for the brake to resist & torque of 800 Nm. (b) Assuming that the value of the design factor $pV = 1\ \mathrm{MPa} \cdot \mathrm{m/s}$ projected bearing area and that the normal operating speed of the drum is 300 rpm, determine the width of brake shoes required